Modeling and model identification of a pressurizer at the Paks Nuclear Power Plant

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Pressurizer
Engineering model

Water energy balance

$$\frac{dU}{dt} = c_p m T_I - c_p m T + K_W (T_W - T) + \sum_{i=1}^{4} W_{HE} \cdot \chi_i$$

Wall energy balance

$$\frac{dU_W}{dt} = K_W (T - T_W) - W_{loss}$$

Constitutive equations

$$\begin{aligned} U &= c_p M T, \\ U_W &= C_{pW} T_W, \end{aligned}$$

Pressurizer
State-space model

Pressure equation

$$p = h(T) = \frac{e^{\varphi(T)}}{100} \quad , \quad \varphi(T) = c_0 + c_1 T + c_2 T^2 + c_3 T^3$$

State-space model:

$$\begin{aligned}\dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}\bar{u} + \bar{E}\bar{d} \\ \bar{x} &= [T \;\; T_W]^T \;\; , \;\; \bar{u} \in \{0, 1, 2, 3, 4\} \;\; , \;\; \bar{d} = [T_I \;\; W_{loss}]^T \\ \\ \bar{y} &= \begin{bmatrix} 1 & 0 \end{bmatrix} \bar{x} \;\; , \;\; \bar{x}_1 = h^{-1}(p)\end{aligned}$$

Pressurizer Parameters

State space equation matrices:

$$\bar{A} = \begin{bmatrix} -\frac{m}{M} - \frac{K_W}{c_p M} & \frac{K_W}{c_p M} \\ \frac{K_W}{C_{pW}} & -\frac{K_W}{C_{pW}} \end{bmatrix}$$

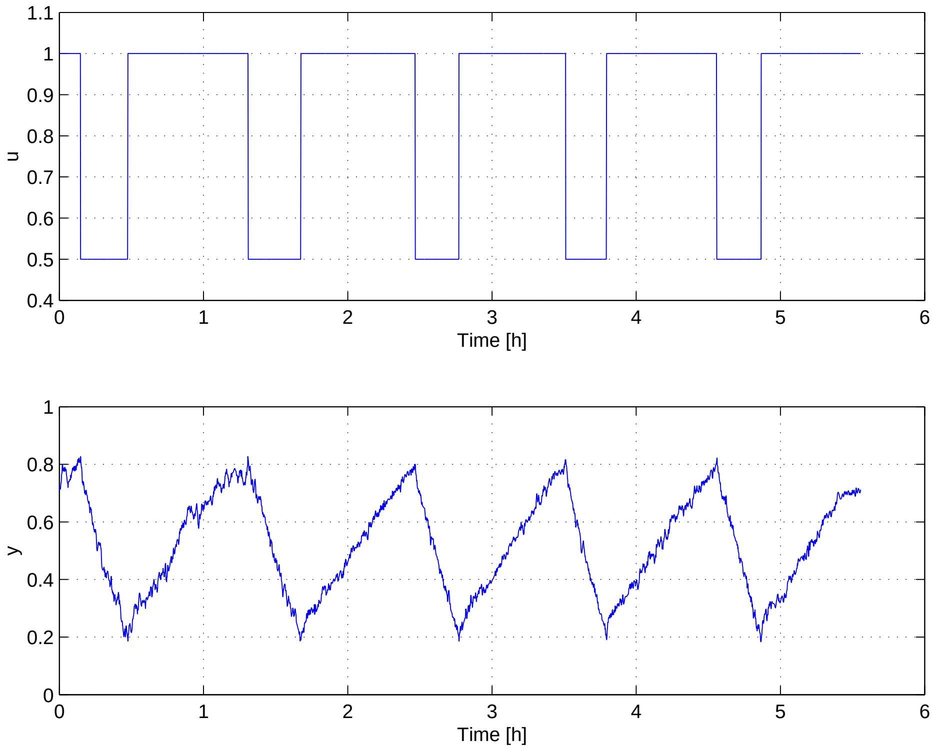
$$\bar{B} = \begin{bmatrix} \frac{W_{HE}}{c_p M} \\ 0 \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} \frac{m}{M} & 0 \\ 0 & \frac{1}{C_{pW}} \end{bmatrix}$$

1. *Parameters with known values*: W_{HE}, c_p, M, m
2. *Parameter with an acceptable initial guess*: W_{loss}
3. *Parameters with completely unknown values*: C_{pW}, K_W.

Identification
Preliminary steps

Scaling: to obtain a dimensionless form

Measured input and output: scaled



Sufficient excitation: by the old "on-off" controller

Identification
Structure validation

ESS method: set of candidate model structures

$$y(k) \;=\; \sum_{i=1}^{n_r} \frac{b_i z^{-1}}{1 + a_i z^{-1}} u(k-d) + \sum_{i=1}^{n_c} \frac{q_{1i} z^{-1} + q_{2i} z^{-2}}{1 + p_{1i} z^{-1} + p_{2i} z^{-2}} u(k-d)$$

Investigated structures (with $d = 0$):

$$n_r = 1, n_c = 0;\; n_r = 2, n_c = 0;\; n_r = 3, n_c = 0$$
$$n_r = 4, n_c = 0;\; n_r = 1, n_c = 1;\; n_r = 1, n_c = 2$$
$$n_r = 3, n_c = 1;\; n_r = 2, n_c = 1$$

Result of model structure validation: physical model structure ($n_r = 2, n_c = 0$) confirmed (M is known)

Identification
Initial step

Criterion functions:

$$V_1(N,\theta) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y(i)-\hat{y}(i))^2}\ ,\ \ V_2(N,\theta) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(p(i)-\hat{p}(i))^2}$$

Parameters to be estimated:

$$\theta = \begin{bmatrix} C_{pW} & K_W & W_{loss} \end{bmatrix}^T\ ,\ \ (x_2(0))$$

Method: Nelder-Mead simplex search

Results:

parameter	lower end (%)	upper end (%)
C_{pW}	93.284	107.46
K_W	94.69	105.983
W_{loss}	99.9432	100.0516

Identification
Iterative procedure for refining

$\theta_1 = [C_{pW}\ K_W\ W_{loss}]^T$ and $\theta_2 = [c_0\ c_1\ c_2\ c_3]^T$

1. i:=0; set the values of $\hat{\theta}_1(0)$ and $\hat{\theta}_2(0)$
2. Using the initial values $\hat{\theta}_2(i)$, estimate θ_2 with fixed $\theta_1 := \hat{\theta}_1(i)$ applying the simplex search method and criterion function $V = V_1 + V_2$; results in $\hat{\theta}_2(i+1)$.
3. Invert the temperature from the pressure measurements using $\hat{\theta}_2(i+1)$.
4. Using the initial values $\hat{\theta}_1(i)$ estimate θ_1 with fixed $\theta_2 := \hat{\theta}_2(i+1)$ applying the simplex search method and criterion function $V = V_1 + V_2$; results in $\hat{\theta}_1(i+1)$.
5. If sufficient convergence in V , then stop, else i:=i+1; go to step (2).

Identification Results

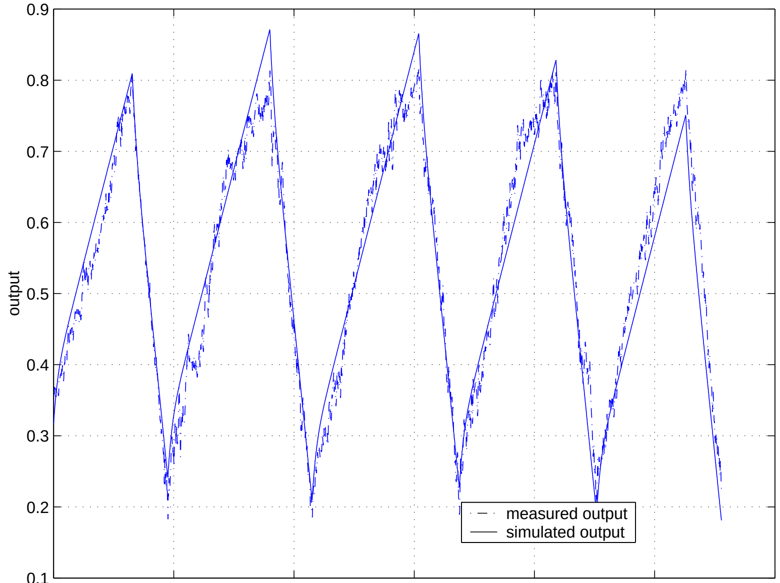
Scaled model matrices:

$$A = 10^{-3}\begin{bmatrix} -4.465 & 4.456 \\ 5.152 & -5.152 \end{bmatrix}, B = \begin{bmatrix} 3.163 \cdot 10^{-3} \\ 0 \end{bmatrix}, E = \begin{bmatrix} 2.927 \cdot 10^{-3} & 0 \\ 0 & -2.03 \cdot 10^{-8} \end{bmatrix}$$

Updated pressure-temperature curve coefficients:

$$c_0 = 0.6397,\ c_1 = 4.893 \cdot 10^{-2}$$
$$c_2 = -9.238 \cdot 10^{-5},\ c_3 = 7.608 \cdot 10^{-8}$$

Model validation



Conclusion

Dynamic model: ***low order***, based on first engineering principles

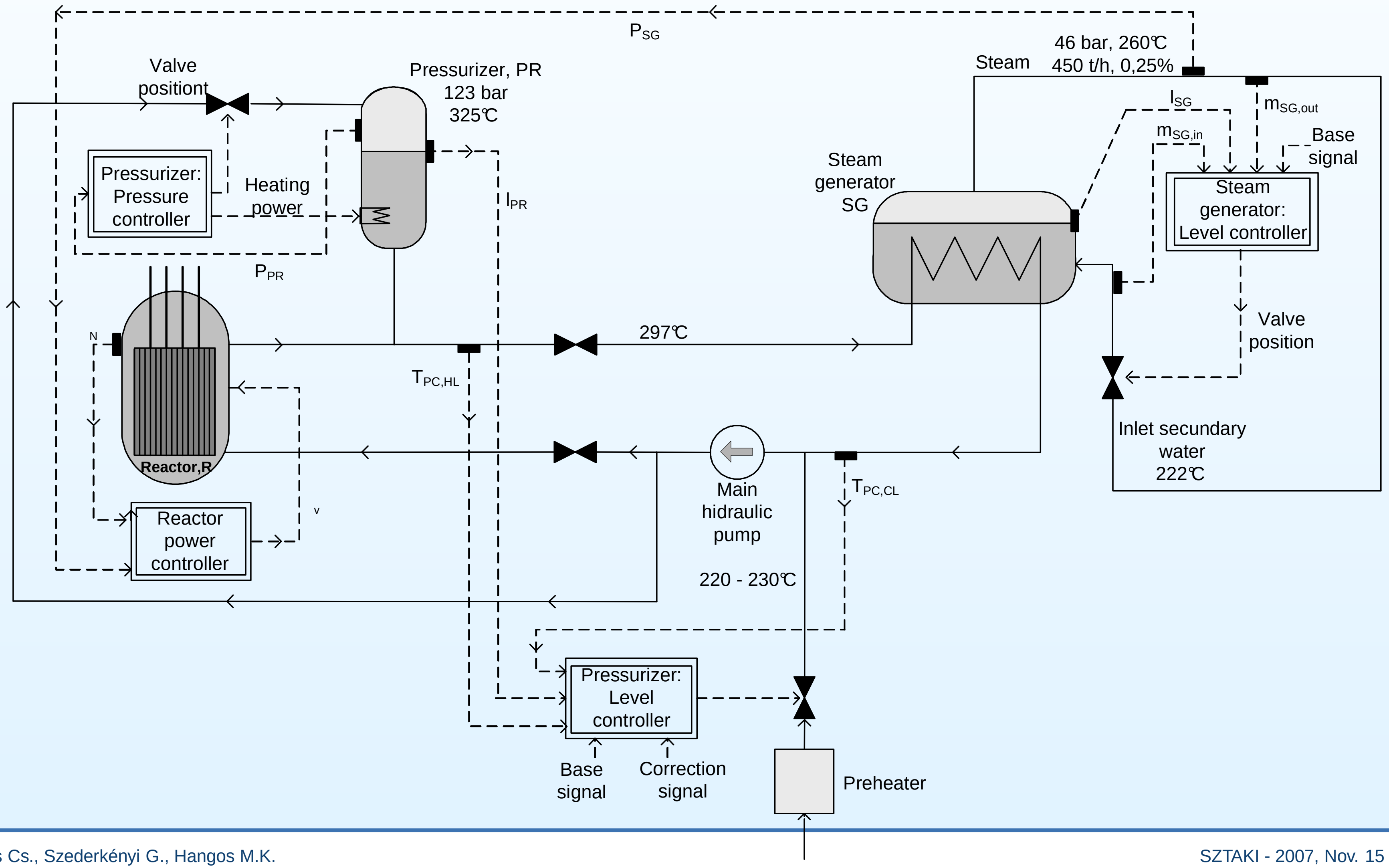
- ***Advantage***: good initial guesses were available
- Model structure validation by ESS method

Parameter estimation: nonlinear in parameters, simplex method

- ***physical parameters***: small number, in the state equations
- pressure-temperature curve parameters: in the output equation
- ***simple iterative scheme*** followed by an initial step

Aim and the physical system

Aim: developing a dynamic model of the primary circuit of VVER plants for controller design purposes. Model has to be minimal and its parameters have to be physical meaning.



Model: State space model

$$\frac{dN}{dt} = \frac{p_1 v^2 + p_2 v + p_3}{\Lambda} N + S \quad (25)$$

$$\frac{dM_{PC}}{dt} = m_{in} - m_{out} \quad (26)$$

$$\frac{dT_{PC}}{dt} = \frac{1}{c_{p,PC} M_{PC}} [c_{p,PC} m_{in} \left(T_{PC,I} - T_{PC}\right) + c_{p,PC} m_{out} 15 + W_R -$$
$$-6 \cdot K_{T,SG,1} \left(T_{PC} - T_W\right)^a - K_{loss,PC} \left(T_{PC} - T_{out,PC}\right)] \quad (27)$$

$$\frac{dM_{SG}}{dt} = m_{SG,in} - m_{SG,out} \quad (28)$$

$$\frac{dT_{SG}}{dt} = \frac{1}{c^L_{p,SG} M_{SG}} [c^L_{p,SG} m_{SG,in} \left(T_{SGSW} - T_{SG}\right) + c^L_{p,SG} m_{SG,out} T_{SG} -$$
$$-m_{SG,out} E_{evap,SG} + K_{T,SG,2} \left(T_W - T_{SG}\right)^b - K_{loss,SG} \left(T_{SG} - T_{out,SG}\right)] \quad (29)$$

$$\frac{dT_W}{dt} = \frac{1}{c_{p,W} M_W} \left(K_{T,SG,1} \left(T_{PC} - T_W\right)^a - K_{T,SG,2} \left(T_W - T_{SG}\right)^b\right) \quad (30)$$

$$\frac{dT_{PR}}{dt} = \frac{1}{c_{p,PR} M_{PR}} [\chi_{m_{PR}>0} c_{p,PC} m_{PR} T_{PC,HL} + \chi_{m_{PR}<0} c_{p,PR} m_{PR} T_{PR} -$$
$$-W_{loss,PR} + W_{heat,PR} - c_{p,PR} m_{PR} T_{PR}] \quad (31)$$

$$W_R = c_\Psi N \quad (32)$$

$$p_{SG} = p^T_*(T_{SG}) \quad (33)$$

$$\ell_{PR} = \frac{1}{A_{PR}} \left(\frac{M_{PC}}{\varphi(T_{PC})} - V^0_{PC}\right) \quad (34)$$

$$p_{PR} = p^T_*(T_{PR}) \quad (35)$$

Model: Variables and parameters

Variables

- *State variables*: N, M_{PC}, T_{PC}, T_{PR}, M_{SG}, T_{SG}, T_W
- *Input variables*: v, m_{in}, $m_{SG,in}$, $W_{heat,PR}$
- *Disturbances*: m_{out}, $m_{SG,out}$, $T_{SG,SW}$, $T_{PC,I}$
- *Output variables*: N (W_R), ℓ_{PR} (M_{PC}), p_{PR}, p_{SG}

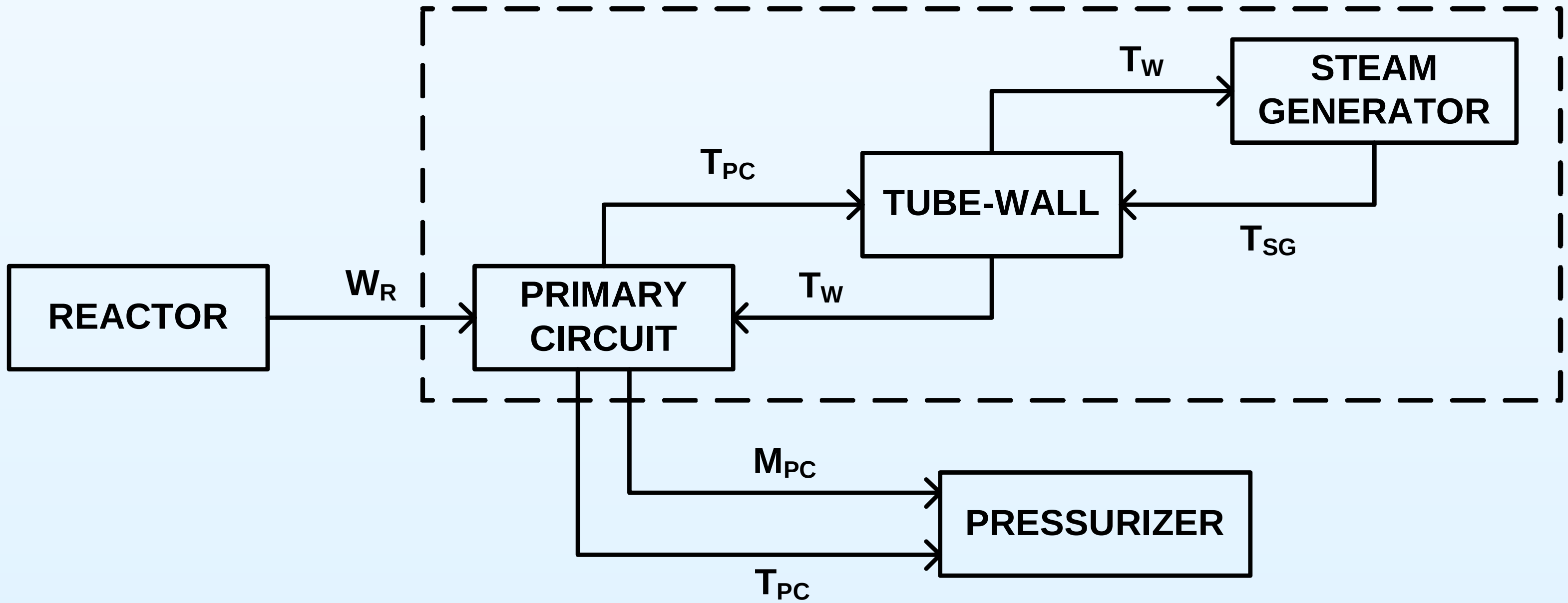
Constants

- Reactor: parameters of nuclear physics: $\frac{1}{\Lambda}$, S; parameters of the control rod: p_1, p_2, p_3; other: c_Ψ.
- Liquid in the primary circuit: $c_{p,PC}$, $K_{T,SG,1}$, $K_{loss,PC}$, a.
- Steam generator: $c^L_{p,SG}$, $K_{T,SG,2}$, $K_{loss,SG}$, b.
- Wall of the steam generator: $c_{p,W} \cdot M_W$.
- Pressurizer: $c_{p,PR}$, $W_{loss,PR}$, V^0_{PC}.

Known constants are $T_{out,PC} = 100\ ^oC$, $T_{out,SG} = 260\ ^oC$ (in case of the unit 1 and 3) and $T_{out,SG} = 259\ ^oC$ (in case of the unit 4)

PE: Decomposition

It is seen from the state equations (25)-(31) that the parameters in the neutron flux balance equation (25) can be estimated independently of the parameters of the other operating units.
Then the coupled equations (26)-(30) describing the dynamics of the liquid in the primary circuit, in the steam generator and the dynamics of the tube-wall of the steam generator form another component.
The third component is the pressurizer that depends on the dynamics of the liquid in the primary circuit.

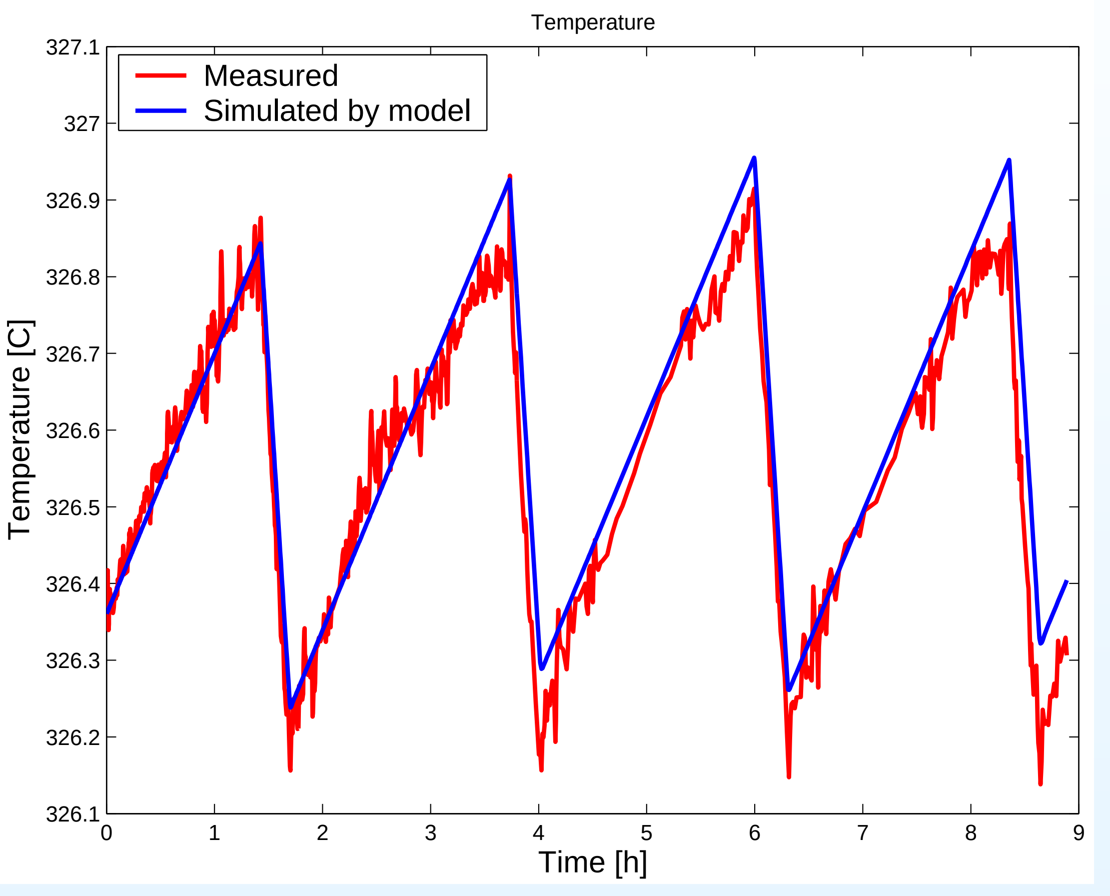


Results: Pressurizer 1

A good quality and long measured data sequence from the Unit computer exists describing the dynamics of pressurizer using the old "on-off type" pressure controller, this data sequence is used for parameter estimation.

Parameter	**Unit**	Old data Time span: 8.88 h
$c_{p,PR}$ $W_{loss,PR}$	$J/kg/K$ W	5886.3 $1.6811 \cdot 10^5$
Error	-	$5.9828 \cdot 10^{-4}$

Results: Pressurizer 2



Results: Quality of estimates 3