Számítógéppel irányított rendszerek elmélete

Gyakorlat - CT-LTI Stabilitás

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Exercise 1

Given the following CT LTI SISO state space model:

$$
\begin{aligned}
\dot{x} &= \begin{bmatrix} -1 & 6 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\
y &= \begin{bmatrix} 2 & -4 \end{bmatrix} x
\end{aligned}
$$

- Compute the transfer function of the system.
- Is the system jointly controllable and observable?
- Is the system asymptotically/BIBO stable?

Solution to Exercise 1 - Transfer function

The SSR is in controller form, therefore

$$H(s) = \frac{2s-4}{s^2+s-6} = \frac{2(s-2)}{(s+3)(s-2)}$$

- The system IS NOT jointly controllable and observable, because there is a common root (+2) of the nominator and the denominator in $H(s)$.
- The system IS NOT asymptotically stable, since there is a pole (+2) with positive real part. However, it is BIBO stable, since the minimal realization with the reduced transfer function

$$\overline{H}(s) = \frac{2}{s+3}$$

  is asymptotically stable.

Exercise 2

We are given the following differential equation:

$$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 2\dot{u}(t) + u(t)$$

1. Is the above system stable (BIBO, or asymptotically)?
2. Give a possible state-space representation of the system!
3. Is the system controllable and/or observable?

Solution to Exercise 2 – 1 and 2

1. Stability:
   The poles (eigenvalues) of the system are:

   $$\lambda_1 = -2, \quad \lambda_2 = -3$$

   so it is asymptotically stable. Asymptotic stability implies BIBO stability, so it is BIBO stable.

2. Possible state space representation
   Solution: controller form:

   $$\begin{aligned} \dot{x} &= \begin{bmatrix} -5 & -6 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} 2 & 1 \end{bmatrix} x \end{aligned}$$

Solution to Exercise 2 – 3

Given the CT LTI SISO input-output model:

$$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 2\dot{u}(t) + u(t)$$

3. Controllability, observability
   Solution:

$$H(s) = \frac{2(s+0.5)}{(s+2)(s+3)} \quad \text{is irreducible,}$$

   i.e. the system is joint controllable and observable.

Exercise 3

Given the following CT LTI SISO system:

$$\dot{x} = \begin{bmatrix} 3 & 0 \\ 5 & p \end{bmatrix} x + \begin{bmatrix} q \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} r & 1 \end{bmatrix} x$$

1. Give the parameter values p, q, r so that the above system is asymptotically stable! (If it is not possible, why?)
2. Give the parameter values p, q, r so that the above system is BIBO stable! (If it is not possible, why?)

Solution to Exercise 3

$$\dot{x} = \begin{bmatrix} 3 & 0 \\ 5 & p \end{bmatrix} x + \begin{bmatrix} q \\ 1 \end{bmatrix} u \;, \quad y = \begin{bmatrix} r & 1 \end{bmatrix} x$$

1. Give the parameter values p, q, r so that the above system is asymptotically stable! (If it is not possible, why?)
   Solution: Not possible, since $\lambda_1 = 3$. (The state matrix is lower triangular!)
2. Give the parameter values p, q, r so that the above system is BIBO stable! (If it is not possible, why?)
   Solution: From the condition

$$\int_0^\infty |h(t)| dt < \infty$$

and using

$$h(t) = Ce^{At}B$$

$p < 0, \quad q = 0, \quad r = 0.$

Exercise 4

Given the following CT LTI SISO system:

$$\dot{x} = \begin{bmatrix} -3 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 3 & 2 \end{bmatrix} x$$

$$Q = \begin{bmatrix} 12 & 2 \\ 2 & 4 \end{bmatrix}$$

- Is it possible to find a $P > 0$ symmetric matrix for any $Q > 0$ symmetric matrix such that $A^T P + PA = -Q$?
  Solution: The eigenvalues of A are $\lambda_1 = -1$, and $\lambda_2 = -3$, i.e. A is a stability matrix, so according to the *Lyapunov criterion for LTI systems*, it is possible.

Homework Exercise

Given the following CT LTI SISO system:

$$\dot{x} = \begin{bmatrix} -3 & 0 \\ 2 & p \end{bmatrix} x + \begin{bmatrix} q \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} r & 2 \end{bmatrix} x$$

- Give the values of parameters p, q and r so that the system is asymptotically stable and minimal!