Számítógéppel irányított rendszerek elmélete

Gyakorlat - Megfigyelhetőség és Irányíthatóság

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Exercise 1

Given the CT LTI system:

$$
\begin{aligned}
\dot{y}_1 + y_3 &= u_1 - u_2 \\
\ddot{y}_1 + \dot{y}_1 &= y_2 + u_2 \\
\dot{y}_3 + u_1 - u_2 &= 0 \\
\dot{y}_2 + y_1 + y_3 &= u_1
\end{aligned}
$$

- Give a possible state space representation of the system defined by the above equations.
- Give the controllability matrix of the model! Is it controllable?
- Give the observability matrix of the model! Is it observable?

Solution to Exercise 1 - State space model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ -1 & 1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Solution to Exercise 1 - Controllability

$$\mathcal{C} = \begin{bmatrix} 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 1 & 0 & 0 & -1 & 1 \end{bmatrix}$$

It is of full rank, so the model is controllable.

Exercise 2

Given the CT LTI state space model:

$$\dot{x} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} x + \begin{bmatrix} 5 \\ 6 \end{bmatrix} u$$

$$y = \begin{bmatrix} 7 & 8 \end{bmatrix} x$$

1. Is it controllable?
2. Is it observable?
3. Give the transfer function of the system!

Solution to Exercise 2 – 1 and 2

1. Controllability:

$$\mathcal{C} = \begin{bmatrix} 5 & 17 \\ 6 & 39 \end{bmatrix} \quad \Rightarrow \quad det(\mathcal{C}) \neq 0$$

So it is controllable.

2. Observability

$$\mathcal{O} = \begin{bmatrix} 7 & 8 \\ 31 & 46 \end{bmatrix} \quad \Rightarrow \quad det(\mathcal{O}) \neq 0$$

So it is observable.

Solution to Exercise 2 – 3

$$\dot{x} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} x + \begin{bmatrix} 5 \\ 6 \end{bmatrix} u$$

$$y = \begin{bmatrix} 7 & 8 \end{bmatrix} x$$

3. Transfer function

$$H(s) = C(sI - A)^{-1}B = \frac{83s + 16}{s^2 - 5s - 2}$$

Exercise 3

Given the CT LTI state space model:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 3 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 2 \end{bmatrix} x$$

1. Give the state transformation T_D which diagonalizes the system!
2. Give the matrices A_D, B_D, and C_D of the diagonal state space model!
3. Is the system controllable and/or observable?
4. Give the transfer function of the system!

Solution to Exercise 2

1. T_D is the inverse of the matrix constructed from the eigenvectors:

$$T_D = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

2. Diagonal realization

$$A_D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \ , \ B_D = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \ , \ C_D = \begin{bmatrix} 2 & -2 \end{bmatrix}$$

3. Controllable and observable because

$$\lambda_1 \neq \lambda_2, b_i \neq 0,\ c_i \neq 0,\ i = 1, 2.$$

4. Transfer function

$$H(s) = \frac{4}{s-1} - \frac{2}{s+1}$$

Homework Exercise

Given the CT LTI state space model:

$$\dot{x} = \begin{bmatrix} p & 1 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} q \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} x$$

1. Give the values of parameters p and q so that the system is controllable and NOT observable!